**OPTIMIZATION FINAL REPORT**

**BRIEF OVERVIEW**

This report is intended to provide analysis and guidance to management at XYZ Manufacturing on the proposed production and distribution of its new product, the flugel. With two plants and corresponding production lines, four warehouses, and eight retail centers, XYZ Manufacturing is well positioned to minimize costs to $146,620.47 as they successfully fulfill the forecasted demand in the next 10 years.

**RESULTS AND RECOMMENDATIONS**

Objective: minimized costs to $146,620.47

1. Open Condition – whether a production line at a plant is operating in a given year

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Plant** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

In order to meet demand, plant 5 is scheduled to have its production line constructed in year 1 and operating each year. Additionally, plant 2 will have constructed its production line in year 3 and operate each year thereafter.

1. Construction Condition – years in which production lines are constructed

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Plant** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Construction costs (and by default, re-opening costs) will be incurred in years 1 and 3 at plants 2 and 5 for constructing production lines.

1. Shutdown Condition – whether a plant will shut down a production line for flugels and convert it to another purpose

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Plant** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

No production line is planned to shut down and be re-converted over the next 10 years.

1. Re-opening Condition – tracks re-opening costs for production lines which were revived following shut-down

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Plant** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total Costs | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

As no production lines are planned to shut down, no re-opening costs will be incurred.

1. Total Flugel production

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Plant** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 12205 | 11508 | 1 | 12000 | 12000 | 12000 | 12000 | 8827 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 12766 | 12766 | 12766 | 12766 | 12766 | 12766 | 12766 | 12766 | 12766 | 12766 |
| Total | | 12766 | 12766 | 24971 | 24274 | 12767 | 24766 | 24766 | 24766 | 24766 | 21593 |

1. Warehouse Flows – inventory flows in units of flugels from the 5 plants into the 4 warehouses





















1. Warehouse Flows Outbound – inventory flows of units of flugels from the 4 warehouses to the 8 retail centers





















1. Stock Variables - units of flugel saved as stock in each warehouse in each year

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **Year** | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Warehouse** | 1 | 0 | 840 | 840 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 880 |
| 2 | 0 | 0 | 3863 | 4137 | 3863 | 4137 | 3863 | 3068 | 4932 | 0 |
| 3 | 0 | 1772 | 3532 | 4468 | 3532 | 3532 | 3532 | 3173 | 3173 | 0 |
| 4 | 1726 | 0 | 1148 | 6852 | 1148 | 3640 | 4360 | 3640 | 702 | 0 |

1. Alloy – lbs required for units of flugel each year and costs

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Alloy/pounds** | | **Year** | | | | | | | | | |
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **Plant** | **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 5736 | 5409 | 0 | 5640 | 5640 | 5640 | 5640 | 4149 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 6000 | 6000 | 6000 | 6000 | 6000 | 6000 | 6000 | 6000 | 6000 | 6000 |
| **Cost Price/1000$** | | **120** | **124** | **249** | **249** | **135** | **270** | **278** | **286** | **295** | **265** |

1. Widget Sub-assemblies

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Widget** | | **Year** | | | | | | | | | |
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **Plant** | **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 36615 | 34524 | 0 | 36000 | 36000 | 36000 | 36000 | 26482 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 38298 | 38298 | 38298 | 38298 | 38298 | 38298 | 38298 | 38298 | 38298 | 38298 |
| **Cost\_Price/1000$** | | **4866** | **5012** | **9823** | **9844** | **5476** | **10649** | **10968** | **11297** | **11636** | **10495** |

**Recommendation and Sensitivity Analysis:**

1. Changing Capacity Constraints for Plants
   1. If the original unit capacities for each plant were increased/decreased by 10% with all else remaining the same, then the output for the objective function would be as follow:



When the unit capacity for plants j is increased by 10%, the objective output is reduced by $1,674, which positively helps in minimizing costs associated with the production of Flugels (highlighted in green). However, when unit capacity is decreased by 10% the objective output is understandably increased by $735, which negatively increases the costs associated with production (highlighted in red).

1. Altering Material Constraints
   1. If the original maximum acquisition of alloy in pounds per year at each plant were increased/decreased by 10% with all else remaining the same, then the output for the objective function would be as follow:



When the max acquisition constraint on the pounds alloy required for production at plant j is increased by 10%, the objective output is reduced by $1,832, which positively helps in minimizing costs associated with the production of Flugels. However, if the max acquisition constraint is decreased by 10%, the objective output is increased by $2,068, which negatively increases the costs associated with production.

1. Altering Distribution Constraints
   1. If the original maximum flow in and flow out constraints in units per year at each plant were increased/decreased by 10% with all else remaining the same, then the output for the objective function would be as follow:



When the max flow in flow out constraint on the warehouses is increased by 10%, the objective output is reduced by $272.23, which positively helps in minimizing costs associated with the production of Flugels. However, if the max acquisition constraint is decreased by 10%, the objective output is increased by $94.85, which slightly negatively increases the costs associated with production.

1. Further recommendation

For those constraints above, Material Constraints appear to be most sensitive to our cost, increase the material limit maybe most effective way to reduce our total cost. Distribution Constraint have less influence on our cost, it may not be taken in the first place when minimizing the cost.

Adjusting certain constraints associated with the production and distribution of Flugels can help understand how different values under certain circumstances effect the long-range planning. The above calculations are just a few examples of sensitivity analysis, and could be further extrapolated beyond what has been displayed.

**APPENDICES : INFORMATION AND MATHEMATICAL MODEL**

**DATA:**

(see 3-final data)

**PYTHON CODE:**

(See 2-Flugel-gurobi.py)

**MATHEMATICAL MODEL:**

Indexed Sets:

i=year#(1,2,3,4,5,6,7,8,9,10)

j = plant # (1,2,3,4,5)

k = warehouse # (1,2,3,4)

r = retail center # (1,2,3,4,5,6,7,8)

**TOTAL Objective=**Minimize total cost of these parts add together below:

1.cost for product line= + + +

2.Shipping Costs=plant to warehouse + warehouse to retail center= ∑𝑖 ∑𝑗 pl\_to\_wh10𝑖𝑗kWF𝑖𝑗k + ∑𝑗 ∑𝑘 wh\_to\_cust10ikrWFOi𝑘r

3.Material cost=Alloy costs + Widget subassemblies costs=0(α1) + 9000\* Widget i \*(α2) + (9000\*Widget i+39000\* Widgetcheap i )\*(α3) +4.7\*Fij\* alloy i

**Part 1 .PRODUCTION LINE:**

(i=year, j=plant, k=warehouse, r=retailcenter)

**Objective（part1）:**

cost for product line= + + +

**Data:**

Capacity j = capacity at plant j (units)

Construction\_cost10 ij=Construction cost for product line

Operating\_cost10 ij=annual operating cost for product line

Reopen\_cost10 ij=reopen cost for product line

Shutdown\_cost10 ij=shut down cost for product line

**Variable:**

Construction ij=Construction cost for product line happen or not(binary)

openvar ij=annual operating cost for product line happen or not(binary)

Reopen ij=reopen cost for product line happen or not(binary)

Shutdown ij=shut down cost for product line happen or not(binary)

**Constraints:**

Fij\*Operatingij=WFijk {for newly constructed}

Construction1j+ Construction 2j+ Construction 3j…+ Construction 10j<=1 {for a plant construction can happen once at most during 10 years}

Openvar1j = Construction1j

Openvar2j <= Construction1j + Construction2j

Openvar3j <= Construction1j + Construction2j + Construction3j

Openvar4j <= Construction1j + Construction2j + Construction3j + Construction4j

Openvar5j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j

Openvar6j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j + Construction6j

Openvar7j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j + Construction6j+ Construction7j

Openvar8j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j + Construction6j+ Construction7j + Construction8j

Openvar9j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j + Construction6j+ Construction7j + Construction8j + Construction9j

Openvar10j <= Construction1j + Construction2j + Construction3j + Construction4j+ Construction5j + Construction6j+ Construction7j + Construction8j + Construction9j + Construction10j

{shutdown}

Shutdown1j==0 {shutdown cannot happen in year 1}

Openvarij – Openvar(i+1)j <= Shutdown(i+1)j #{from year 1 to 10}

{Reopening: cannot start until Year 3.}

reopen0j==0

reopen1j==0

Openvar(i-1)j – Openvarij <= Reopenij+ Constructionij

{from year 3 to year 10} i=2-10

Reopenij+ Constructionij <= 1{from year 3 to year 10}i=2-10

# reopen and construction cannot happen in same year

reopen[i,j]+construction[i,j])<=1 for each I and each j

#open and shutdown cannot happen in same year

openvar[i,j]+shutdown[i,j])<=1 for each I and each j

# reopen cannot exceeds 4 times for one plant in 10 years

Reopen1j+ Reopen2j +…+ Reopen10j <=4

#reopen cannot happen if no shutdown

Reopen1j+ Reopen2j +…+ Reopen10j <= shutdown1j+ shutdown 2j +…+ shutdown10j

**Part 2. WAREHOUSE AND SHIPPING:**

**Objective（part2）**:

∑𝑖 ∑𝑗 pl\_to\_wh10𝑖𝑗kWF𝑖𝑗k + ∑𝑗 ∑𝑘 wh\_to\_cust10ikrWFOi𝑘r

year=[0,1,2,3,4,5,6,7,8,9]#i=10

plant=[0,1,2,3,4]#j=5

warehouse=[0,1,2,3]#k=4

retailcenter= [0,1,2,3,4,5,6,7]#r=8

**Data:**

pl\_to\_wh10ijk= cost of shipping one flugel from plant j to warehouse k

wh\_to\_cust10ikr= cost of shipping one flugel from warehouse k to retail center r

Demandin10years ik = demand for retail center k in year i (units)

**Variables:**

WFijk = units of shipping one flugel from plant j to warehouse k(warehouse flow in)

WFOikr = units of shipping one flugel from warehouse k to retail center r (warehouse flow out)

Sik=units of flugel saved as stock in the warehouse k in year i

Fij= units of flugel produced in plant j in year i

**Constraints:**

#Capacitycontrol

Fvars[i,j]<= (Capacity[j]\*openvar[i,j]+Capacity[j]\*construction[i,j]+Capacity[j]\*reopen[i,j]) (for i in year for j in plant )

#if close, should not produce any.( 9000000000 is a random big enough figure)

Fvars[i,j]<=9000000000\*openvar[i,j]

( for i in year for j in plant for k in warehouse)

#every year flow into warehouse should equals to each plant produce

Fvars[i,j]==WFvars.sum(i,j,'\*')

(for i in year for j in plant for k in warehouse)

S10k==0{last year inventory will be 0}

Sum WFOikr== sum Demandin10years ik { meet Demand}

Fij==sum(WF𝑖𝑗k ) {for each plant j all flow into 4 warehouse}

WFI 1jk = WFO1kr+S1k { first year do not have inventory from before}

WFI ijk+S(i-1)k = WFOikr+Sik {for each warehouse}

Sik<=12000{maximum for a warehouse to handle}

WFijk <=12000{each warehouse max flow in}

WFOikr <=12000{each warehouse max flow out}

(S(i-1)k+ Sik)/2<=4000{average inventory in year i to be no more than 4000 items for a warehouse. }

#first year ave inventory not exceed 4000

svars[0,k]<=4000\*2 ( for k in warehouse)

**Part 3. MATERIAL:**

**Objective（part3）:**

Widget subassemblies cost= 0(α1) + 9000\* Widget i \*(α2) + (9000\*Widget i+39000\* Widgetcheap i )\*(α3) {at each plant I each year}

Alloy cost= 4.7\*Fij\* alloy i {at each plant each year}

**Data:**

alloy i= {0.02 unit price at year i, increased by 3% each year}

Widget i = {0.15 unit price at year i, increased by 3% each year}

Widgetcheap i= {0.12 unit price at year i, increased by 3% each year when exceed 9000}

**Variable:**

Alpha1ij >= 0 {control the section choose of widget buying}

Alpha1 ij>= 0 {control the section choose of widget buying}

Alpha1ij >= 0 {control the section choose of widget buying}

W1ij = binary {control the section choose of widget costing}

W2ij = binary {control the section choose of widget costing}

**Constraints:**

Fij\*4.7<=60000 {max lbs of alloy per year at each plant i}

Fij=0(α1ij) + 3000(α2ij) + 16000 (α3ij) {for each plant in each year}

{For any I and j:}

α1 ij+ α2ij + α3ij = 1

W1 ij + W2 ij = 1

α1 ij <= W1 ij

α2 ij <= W1 ij + W2 ij

α3 ij <= W2 ij

**DATA:**

(see 3-final data)

**PYTHON CODE:**

(See 2-Flugel-gurobi.py)